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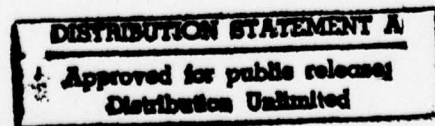DDC

COMPUTING STATIONARY POINTS, AGAIN

TECHNICAL REPORT

B. CURTIS EAVES

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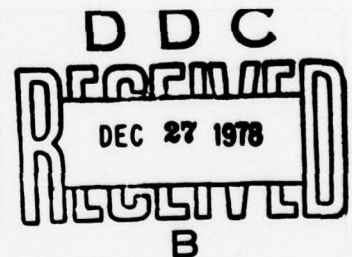
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COMPUTING STATIONARY POINTS, AGAIN

by

B. Curtis Eaves

1. Introduction and Abstract

Given a nonempty set $\mathcal{X}$ in R^n of form $\{x \geq 0 : Ax \leq a\}$ with an affine function, $\beta(x) = Cx + c$ from $\mathcal{X}$ to R^n, we consider (existence and) computation, in a finite number of steps, of a stationary point. A point x^* in $\mathcal{X}$ is defined to be a stationary point of $(\mathcal{X}, \beta)$ if $x \cdot \beta(x^*) \geq x^* \cdot \beta(x^*)$ for all x in $\mathcal{X}$. The existence and computation of stationary points is, in particular, central to the solution of certain quadratic programs, matrix games, and economic equilibrium problems. Computing a stationary point of $(\mathcal{X}, \beta)$ is equivalent to solving the linear complementary problem,

$$\begin{pmatrix} C & A^T \\ -A & 0 \end{pmatrix} \begin{pmatrix} x \\ \lambda \end{pmatrix} + \begin{pmatrix} c \\ a \end{pmatrix} = \begin{pmatrix} \mu \\ s \end{pmatrix}$$

(1)

$$x \geq 0 \qquad \lambda \geq 0 \qquad \mu \geq 0 \qquad s \geq 0$$

$$x \cdot \mu = \lambda \cdot s = 0$$

in the sense that if x^* is a stationary point, then there is an (x^*, λ, μ, s) solving (1), and reversely, if (x, λ, μ, s) solves (1), then x is a stationary point.

-1-

Towards describing our result, let x^0 be any extreme point of $\mathcal{X}$. Consequently, there is a submatrix (L, ℓ) of n rows of the matrix.

$$\begin{pmatrix} I & 0 \\ -A & -a \end{pmatrix}$$

such that $Lx^0 = \ell$ and L is nonsingular. For any such L let $p = L^T \tilde{p}$ where $\tilde{p}$ is any positive vector. We shall use the family of stationary problems $(\mathcal{X}, \mathcal{E}_\theta)$ where $\mathcal{E}_\theta(x) \triangleq \mathcal{E}(x) + \theta p$ and $\theta \geq 0$ in order to solve the stationary problem $(\mathcal{X}, \mathcal{E}_0) = (\mathcal{X}, \mathcal{E})$. Observe that x^0 is a stationary point of $(\mathcal{X}, \mathcal{E}_\theta)$ for all sufficiently large θ; let θ^0 be the smallest such θ.

Define a piecewise linear path to be a function from $[0, +\infty)$ to R^k that is affine on each element of a finite closed cover of $[0, +\infty)$. The following theorem captures the principal result.

Theorem: Lemke's algorithm computes a piecewise linear path (X, Θ) such that $X(0) = x^0$, $\Theta(0) = \theta^0$, $X(t)$ is a stationary point of $(\mathcal{X}, \mathcal{E}_{\Theta(t)})$ for all t, and either $\Theta(t) = 0$ for some t or $X(t)$ tends to infinity. ⊠

In our previous study [2] a similar result was developed, however, there the algorithm operated by perturbing $\mathcal{X}$ rather than $\mathcal{E}$. There $\mathcal{X}$ was of form $\{x : Ax \leq a\}$, $\mathcal{X}$ was not required to have an extreme point, and the path could be initiated anywhere in $\mathcal{X}$. Nevertheless, the present scheme should be more effective if applicable, namely, if $\mathcal{X}$ lies in the nonnegative orthant and the algorithm is to be initiated at an extreme point.

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Although Mylander [4; Ch 4] was studying quadratic programming
(C was assumed symetric) under a nondegeneracy condition and from a
perspective of parametrizing the feasible region ($\mathcal{X}$ was deformed via
$\mathcal{X} \cap \{e \cdot x \leq \theta\}$) rather than the objective, much, or perhaps all, of the
Theorem can be extracted from his proof. In [1; § 11] it is shown that
Lemke's algorithm can be used to compute $(x, \bar{x}, \theta, \bar{\theta})$ so that $x + t\bar{x}$
is a stationary point of $(\mathcal{X}, \mathcal{C}(\theta + t\bar{\theta}))$ for all $t \geq 0$ where $\theta = 0$
or $\bar{x} \neq 0$; here both $\mathcal{X}$ and $\mathcal{C}$ were parametrized.

2. A Special Case

In this section we assume x^0 is the origin and $(L, \ell) = (I, 0)$, and under these conditions we prove the Theorem. Let p in R^n be any positive vector and we apply Lemke's algorithm (see [1] or [3]) to the augmented linear complementary problem

$$\begin{pmatrix} C & A^T \\ -A & 0 \end{pmatrix} \begin{pmatrix} x \\ \lambda \end{pmatrix} + \begin{pmatrix} c \\ a \end{pmatrix} + \begin{pmatrix} p \\ 0 \end{pmatrix} \theta = \begin{pmatrix} \mu \\ s \end{pmatrix}$$

(2)

$$x \geq 0, \quad \lambda \geq 0, \quad \mu \geq 0, \quad s \geq 0, \quad \theta \geq 0$$

$$x \cdot \mu = \lambda \cdot s = 0 .$$

Conceptually the first step of the algorithm is to perturb c and a to $c_\epsilon = c + (\epsilon, \epsilon^2, \ldots, \epsilon^n)$ and $a_\epsilon = a + (\epsilon^{n+1}, \ldots, \epsilon^{n+m})$, respectively, where ϵ is a positive infinitesmal; we obtain the perturbed augmented linear complementary problem:

$$\begin{pmatrix} C & A^T \\ -A & 0 \end{pmatrix} \begin{pmatrix} x \\ \lambda \end{pmatrix} + \begin{pmatrix} c_\epsilon \\ a_\epsilon \end{pmatrix} + \begin{pmatrix} p \\ 0 \end{pmatrix} \theta = \begin{pmatrix} \mu \\ s \end{pmatrix}$$

(2,ϵ)

$$x \geq 0, \quad \lambda \geq 0, \quad \mu \geq 0, \quad s \geq 0, \quad \theta \geq 0$$

$$x \cdot \mu = \lambda \cdot s = 0$$

Clearly $(x, \lambda, \mu, s, \theta) = (0, 0, c_\epsilon + \theta p, a_\epsilon, \theta)$ for all sufficiently large θ forms a ray of solutions to (2,ϵ); this ray is referred to as the primary ray.

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The initial solution $(x_\epsilon^0, \lambda_\epsilon^0, \mu_\epsilon^0, s_\epsilon^0, \theta_\epsilon^0)$ is generated by selecting the element of the primary ray with the smallest possible θ. Henceforth we assume that θ_ϵ^0 is positive for if $\theta_\epsilon^0 = 0$, the path of the Theorem is trivially obtained by setting $(X(t), \Theta(t)) = (0, 0)$ for all t. Beginning with the initial solution, Lemke's algorithm complementary pivots and generates a sequence of solutions $(x_\epsilon^i, \lambda_\epsilon^i, \mu_\epsilon^i, s_\epsilon^i, \theta_\epsilon^i)$ $i = 1, \ldots, k$ for $(2,\epsilon)$. Each component of each solution is a poly-nomial of form $\sum_0^{n+m} f_i \epsilon^i$. The algorithm terminates if $\theta_\epsilon^k = 0$ or if a ray is encountered. Such a ray is referred to as the secondary ray and has at most one point in common with the primary ray.

By x_0^t we denote the quantity x_ϵ^t where ϵ has been set to zero, etc. First let us define the function $(X, \Lambda, M, S, \theta)$ on $[0, k]$ by setting $(X, \Lambda, M, S, \theta)(t) = (x_0^t, \lambda_0^t, \mu_0^t, s_0^t, \theta_0^t)$ for $t = 0, 1, \ldots, k$ and by extending it affinely on $[i, i + 1]$ for $i = 0, \ldots, k - 1$.

If the algorithm terminates with $\theta_\epsilon^k = 0$ we obtain a path of solutions to (2) with $\Theta(k) = 0$ by extending $(X, \Lambda, M, S, \theta)$ con-stantly on $[k, +\infty)$, and we obtain, in particular, the path (X, θ) of the Theorem.

So now let us assume the algorithm terminates on a secondary ray. This ray of solutions to $(2,\epsilon)$ will have the form

$$(x_\epsilon^k, \lambda_\epsilon^k, \mu_\epsilon^k, s_\epsilon^k, \theta_\epsilon^k) + t\,(\bar{x}, \bar{\lambda}, \bar{\mu}, \bar{s}, \bar{\theta})$$

with $t \geq 0$ where $(\bar{x}, \bar{\lambda}, \bar{\mu}, \bar{s}, \bar{\theta}) \neq 0$, $(x_\epsilon^k, \lambda_\epsilon^k, \bar{x}, \bar{\lambda}) \neq 0$, and

$$(3) \qquad \begin{pmatrix} C & A^T \\ -A & 0 \end{pmatrix} \begin{pmatrix} \bar{x} \\ \bar{\lambda} \end{pmatrix} + \begin{pmatrix} p \\ 0 \end{pmatrix} \bar{\theta} = \begin{pmatrix} \bar{\mu} \\ \bar{s} \end{pmatrix} .$$

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Our task is now to show that $\bar{x} \neq 0$. Suppose $\bar{x} = 0$ and $x_\epsilon^k = 0$, then $s_\epsilon^k > 0$, then, using $\lambda \cdot s = 0$, we get $\bar{\lambda} = 0$ and $\lambda_\epsilon^k = 0$ which contradicts a property of a secondary ray. So now let us suppose that $\bar{x} = 0$ and $x_\epsilon^k \neq 0$. We have $\bar{s} = 0$, $\bar{\lambda} \cdot Ax_\epsilon^k + \bar{\lambda} \cdot s_\epsilon^k = \bar{\lambda} \cdot a_\epsilon$, and $x_\epsilon^k \cdot \bar{\mu} - x_\epsilon^k \cdot C\bar{x} - x_\epsilon^k \cdot A^T\bar{\lambda} = x_\epsilon^k \cdot p\bar{\theta}$. Combining the last two expressions, using $\mu \cdot x = 0$, $s \cdot \lambda = 0$, and $\bar{x} = 0$ we have $\bar{\lambda} \cdot a_\epsilon = x_\epsilon^k = x_\epsilon^k \cdot p\bar{\theta}$. Hence, $\bar{\lambda} = 0$ and $\bar{\theta} = 0$ or $(\bar{x}, \bar{\lambda}, \bar{\mu}, \bar{s}, \bar{\theta}) = 0$ which is again a contradiction. We may conclude that $\bar{x} \neq 0$. A path of solutions to (2) with $X(t)$ tending to infinity is obtained by extending $(X, \Lambda, M, S, \theta)$ to $[0, +\infty)$ by setting

$$(X, \Lambda, M, S, \theta)(t)$$

$$= (x_0^k, \lambda_0^k, \mu_0^k, s_0^k, \theta_0^k)$$

$$+ (t - k)(\bar{x}, \bar{\lambda}, \bar{\mu}, \bar{s}, \bar{\theta})$$

for $t \geq k$. In particular, (X, θ) is the path of the Theorem.

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3. Overview of Transformation

We shall prove the Theorem by transforming the general case into the special case of the previous section. Here we give an overview of the transformation used.

Consider the stationarity problem $(\mathcal{X}, \beta)$ in R^n and let $\mathcal{A}(x) = Lx + \ell$ be a one to one affine transformation from R^n to R^n. Let $\tilde{\mathcal{X}} = \mathcal{A}(\mathcal{X})$ and $\tilde{\beta}(x) = \tilde{C}x + \tilde{c}$ where $\tilde{C} \triangleq L^{-1T} CL^{-1}$, $\tilde{c} \triangleq L^{-1T} c - \tilde{C}\ell$, and T denotes transpose.

Lemma 1: x^* is a stationary point of $(\mathcal{X}, \beta)$, if and only if $\tilde{x}^* = \mathcal{A}(x^*)$ is a stationary point of $(\tilde{\mathcal{X}}, \tilde{\beta})$

Proof: $x \cdot (Cx^* + c) \geq x^* \cdot (Cx^* + c)$ for all x in $\mathcal{X}$, if and only if $(L^{-1}(\tilde{x} - \ell)) \cdot (C(L^{-1}(\tilde{x}* - \ell)) + c) \geq L^{-1}(\tilde{x}^* - \ell)) \cdot (C(L^{-1}(\tilde{x}^* - \ell)) + c$ for all $\tilde{x}$ in $\tilde{\mathcal{X}}$ $\boxtimes$

Note that the lemma does not require any assumptions on $\mathcal{X}$, the subset of R^n. An interesting case of Lemma 1 is obtained when L is orthogonal; namely, when $L^{-1T} = L$.

4. The General Case

Here we prove the Theorem for general $(\mathcal{X}, \beta)$ by using a transformation as described in Section 3 to obtain the special case of Section 2.

For a matrix B we denote by $B_{\cdot\gamma}$ and $B_{\gamma\cdot}$ the submatrix of rows and columns, respectively, indexed by γ; if B is a vector we drop the dot. Let β be a subset of $\{1, \ldots, m+n\}$ of size m and let α be the complement of β where A is $m \times n$.

Given the extreme point x^0 of $\mathcal{X}$ let $s^0 = a - Ax^0$. We say that β is a basis index yielding (x^0, s^0) if $(A, I)_{\cdot\beta}$ has an inverse and

$$
\begin{pmatrix} x^0 \\ s^0 \end{pmatrix}_\alpha = 0 \qquad \begin{pmatrix} x^0 \\ s^0 \end{pmatrix}_\beta = (A, I)_{\cdot\beta}^{-1} \, a \; .
$$

Observe that β is a basis index yielding (x^0, s^0) if and only if

$$
\begin{pmatrix} I \\ -A \end{pmatrix}_{\alpha\cdot} x^0 = \begin{pmatrix} 0 \\ -a \end{pmatrix}_\alpha
$$

and $\begin{pmatrix} I \\ -A \end{pmatrix}_{\alpha\cdot}$ is nonsingular. Using the simplex method if necessary, select any such α and β.

Define the map $\mathcal{L}$ by $\mathcal{L}(x) = \begin{pmatrix} x \\ s \end{pmatrix}_\alpha = \begin{pmatrix} Ix \\ a - Ax \end{pmatrix}_{\alpha\cdot} = Lx + \ell$ where

$$
L \triangleq \begin{pmatrix} I \\ -A \end{pmatrix}_{\alpha\cdot} \quad \text{and} \quad \ell \triangleq \begin{pmatrix} 0 \\ a \end{pmatrix}_\alpha \; .
$$

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Clearly α is a one to one affine map from χ to $\tilde{\chi}$ where

$$\tilde{\chi} \triangleq \{\tilde{x} \geq 0 : \tilde{A}x \leq a\}$$

$$\tilde{A} \triangleq (A, I)_{\cdot\beta}^{-1} (A, I)_{\cdot\alpha}$$

$$a \triangleq (A, I)_{\cdot\beta}^{-1} a$$

It is important to note that $\tilde{a} \geq 0$ and $\alpha(x^0) = 0$. We form the stationary problem $(\tilde{\chi}, \tilde{\beta})$ by setting

$$\tilde{\beta}(x) \triangleq \tilde{C}x + \tilde{c}$$

$$\tilde{C} \triangleq L^{-1T} CL^{-1}$$

$$\tilde{c} \triangleq L^{-1T} c - \tilde{C}\ell$$

Let $\tilde{p}$ be any positive vector and set $p \triangleq L^T \tilde{p}$. Define $\beta_\theta(x) \triangleq \beta(x) + \theta p$ and $\tilde{\beta}_\theta(x) \triangleq \tilde{\beta}(x) + \theta\tilde{p}$. Of course, Lemma 1 applies to the two systems (χ, β_θ) and $(\tilde{\chi}, \tilde{\beta}_\theta)$; furthermore, $\tilde{a} \geq 0$ and $\tilde{x}^0 \triangleq \alpha(x^0) = 0$, hence, the special case of Section 2 applies to $(\tilde{\chi}, \tilde{\beta})$ with the extreme point $\tilde{x}^0 = 0$.

Now consider the analogue of (2) for $(\tilde{\chi}, \tilde{\beta}_\theta)$, namely

$$(\tilde{2}) \qquad \begin{pmatrix} \tilde{C} & \tilde{A}^T \\ -\tilde{A} & 0 \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{\lambda} \end{pmatrix} + \begin{pmatrix} \tilde{c} \\ \tilde{a} \end{pmatrix} + \begin{pmatrix} \tilde{p} \\ 0 \end{pmatrix} \theta = \begin{pmatrix} \tilde{\mu} \\ \tilde{s} \end{pmatrix}$$

$$\tilde{x} \geq 0 \quad \tilde{\lambda} \geq 0 \quad \tilde{\mu} \geq 0 \quad \tilde{s} \geq 0$$

$$\tilde{x} \cdot \tilde{\mu} = 0 \quad \tilde{\lambda} \cdot \tilde{s} = 0$$

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For this particular type transformation $\mathscr{S}$, Lemma 2 strengthens Lemma 1.

Lemma 2: $(x, \lambda, \mu, s, \theta)$ solves (2), if and only if $(\tilde{x}, \tilde{\lambda}, \tilde{\mu}, \tilde{s}, \theta)$ solves $(\tilde{2})$ where

$$\tilde{x} = \begin{pmatrix} x \\ s \end{pmatrix}_{\alpha} \qquad \tilde{\lambda} = \begin{pmatrix} \mu \\ \lambda \end{pmatrix}_{\beta} \qquad \tilde{\mu} = \begin{pmatrix} \mu \\ \lambda \end{pmatrix}_{\alpha} \qquad \tilde{s} = \begin{pmatrix} x \\ s \end{pmatrix}_{\beta}$$

Proof: We represent (2) with the schema

$$x \geq 0 \qquad \lambda \geq 0 \qquad \mu \geq 0 \qquad s \geq 0 \qquad \theta \geq 0$$

$-C$	$-A^T$	I	0	$-p$	‖	c
A	0	0	I	0	‖	a

$$x \cdot \mu = \lambda \cdot s = 0$$

After regrouping the variables we get the next system which has the same solutions $(x, \lambda, \mu, s, \theta)$.

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$$\begin{pmatrix} x \\ s \end{pmatrix}_\alpha \geq 0 \quad \begin{pmatrix} \mu \\ \lambda \end{pmatrix}_\beta \geq 0 \quad \begin{pmatrix} \mu \\ \lambda \end{pmatrix}_\alpha \geq 0 \quad \begin{pmatrix} x \\ s \end{pmatrix}_\beta \geq 0 \quad \theta \geq 0$$

$(-C,0)_{\cdot\alpha}$	$(I,-A^T)_{\cdot\beta}$	L^T	$(-C,0)_{\cdot\beta}$	$-p$	c
$(A,I)_{\cdot\alpha}$	0	0	$(A,I)_{\cdot\beta}$	0	a

$$\begin{pmatrix} x \\ s \end{pmatrix}_\alpha \cdot \begin{pmatrix} \mu \\ \lambda \end{pmatrix}_\alpha = 0 \qquad \begin{pmatrix} \mu \\ \lambda \end{pmatrix}_\beta \cdot \begin{pmatrix} x \\ s \end{pmatrix}_\beta = 0$$

Now block pivot on $(A, I)_{\cdot\beta}$ to get the next system which also has the same solutions $(x, \lambda, \mu, s, \theta)$.

$$\tilde{x} \geq 0 \qquad \tilde{\lambda} \geq 0 \qquad \tilde{\mu} \geq 0 \qquad \tilde{s} \geq 0 \qquad \theta \geq 0$$

$-(C,0)_{\cdot\alpha}$ $+(C,0)_{\cdot\beta}\tilde{A}$	$(I,-A^T)_{\cdot\beta}$	L^T	0	$-p$	c $+(C,0)_{\cdot\beta}\,\tilde{a}$
$\tilde{A}$	0	0	I	0	$\tilde{a}$

$$\tilde{x} \cdot \tilde{\mu} = 0 \qquad \tilde{\lambda} \cdot \tilde{s} = 0$$

$$\begin{pmatrix} x \\ s \end{pmatrix}_\alpha = \tilde{x} \qquad \begin{pmatrix} \mu \\ \lambda \end{pmatrix}_\beta = \tilde{\lambda} \qquad \begin{pmatrix} \mu \\ \lambda \end{pmatrix}_\alpha = \tilde{\mu} \qquad \begin{pmatrix} x \\ s \end{pmatrix}_\beta = \tilde{s}$$

Premultiply the top row by L^{-1T} to get the schema for (2) which has the same solutions $(\tilde{x}, \tilde{\lambda}, \tilde{\mu}, \tilde{s}, \theta)$

$\tilde{x} \geq 0$ $\tilde{\lambda} \geq 0$ $\tilde{\mu} \geq 0$ $\tilde{s} \geq 0$ $\theta \geq 0$

$-\tilde{C}$	$-\tilde{A}^T$	I	0	$-\tilde{p}$	$\tilde{c}$
$\tilde{A}$	0	0	I	0	$\tilde{a}$

$\tilde{x} \cdot \tilde{\mu} = 0$ $\tilde{\lambda} \cdot \tilde{s} = 0$ ⊠

Thus we apply Lemke's algorithm to $(\tilde{2})$ and according to Section 2 we get a piecewise linear path $(\tilde{X}, \tilde{\Lambda}, \tilde{M}, \tilde{S}, \Theta)$ of solutions with $\tilde{X}(t) = 0$, and either $\theta(t) = 0$ for some t or $\tilde{X}(t)$ tends to infinity as t does. Letting $X(t) = \mathcal{L}^{-1}(\tilde{X}(t))$ the path (X, Θ) is that of the Theorem. Or, in more detail, we can get the path of solutions $(X, \Lambda, M, S, \Theta)$ to (2) by setting

$$\binom{X}{S}_{\alpha} = \tilde{X} \qquad\qquad \binom{X}{S}_{\beta} = \tilde{S}$$

$$\binom{M}{\Lambda}_{\alpha} = \tilde{M} \qquad\qquad \binom{M}{\Lambda}_{\beta} = \tilde{\Lambda} \quad .$$

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5. An Example

Consider the stationarity problem $(\mathcal{X}, \mathcal{C})$ defined by

$$(A,\ a) = \begin{pmatrix} -1 & -1 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{pmatrix}$$

$$(C,\ c) = \begin{pmatrix} 1 & -2 & 0 \\ 1 & -1 & -1 \end{pmatrix}$$
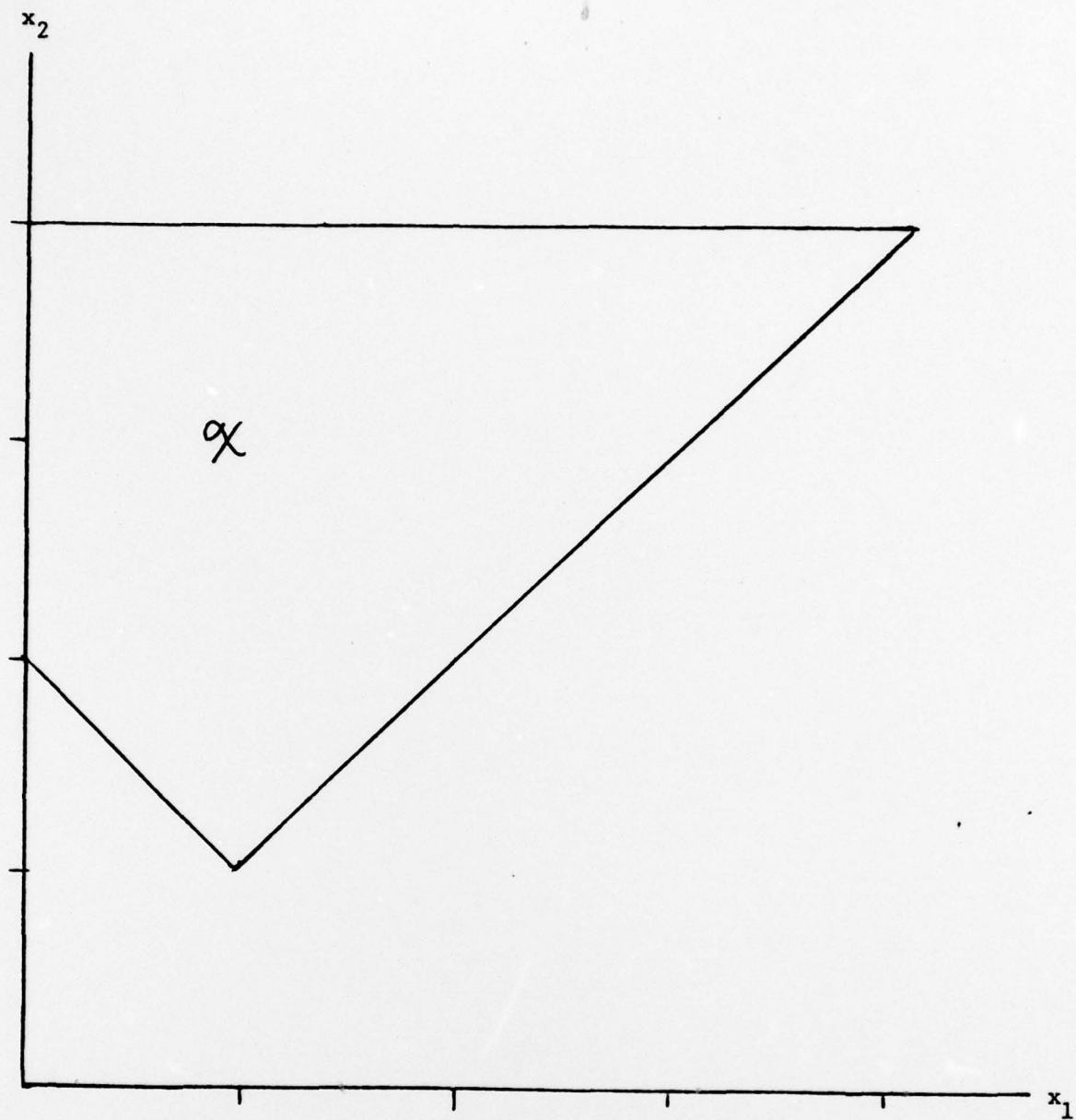
The region is shown in Figure 1.

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Figure 1

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Assuming $x^0 = (0, 2)$ we get $\alpha = \{1, 3\}$, $\beta = \{2, 4, 5\}$, $p = L^T \tilde{p} = (2, 1)$ and $(L, \ell) = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -1 \end{pmatrix}$ where $\tilde{p} = (1, 1)$. The system (2) can be expressed as (4).

(4)

x_1	x_2	λ_1	λ_2	λ_3	μ_1	μ_2	s_1	s_2	s_3	θ	
-1	2	1	-1	0	1	0	0	0	0	-2	0
-1	1	1	1	-1	0	1	0	0	0	-1	-1
-1	-1	0	0	0	0	0	1	0	0	0	-1
1	-1	0	0	0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	0	0	1	0	4

The algorithm can be executed by pivoting sequentially on this matrix of (4) at positions (2, 3), (3, 2), (2, 11), (5, 8), (1, 5), (1, 1), (4, 10), (4, 4), and (2, 5). The path (X, Θ) generated is defined by

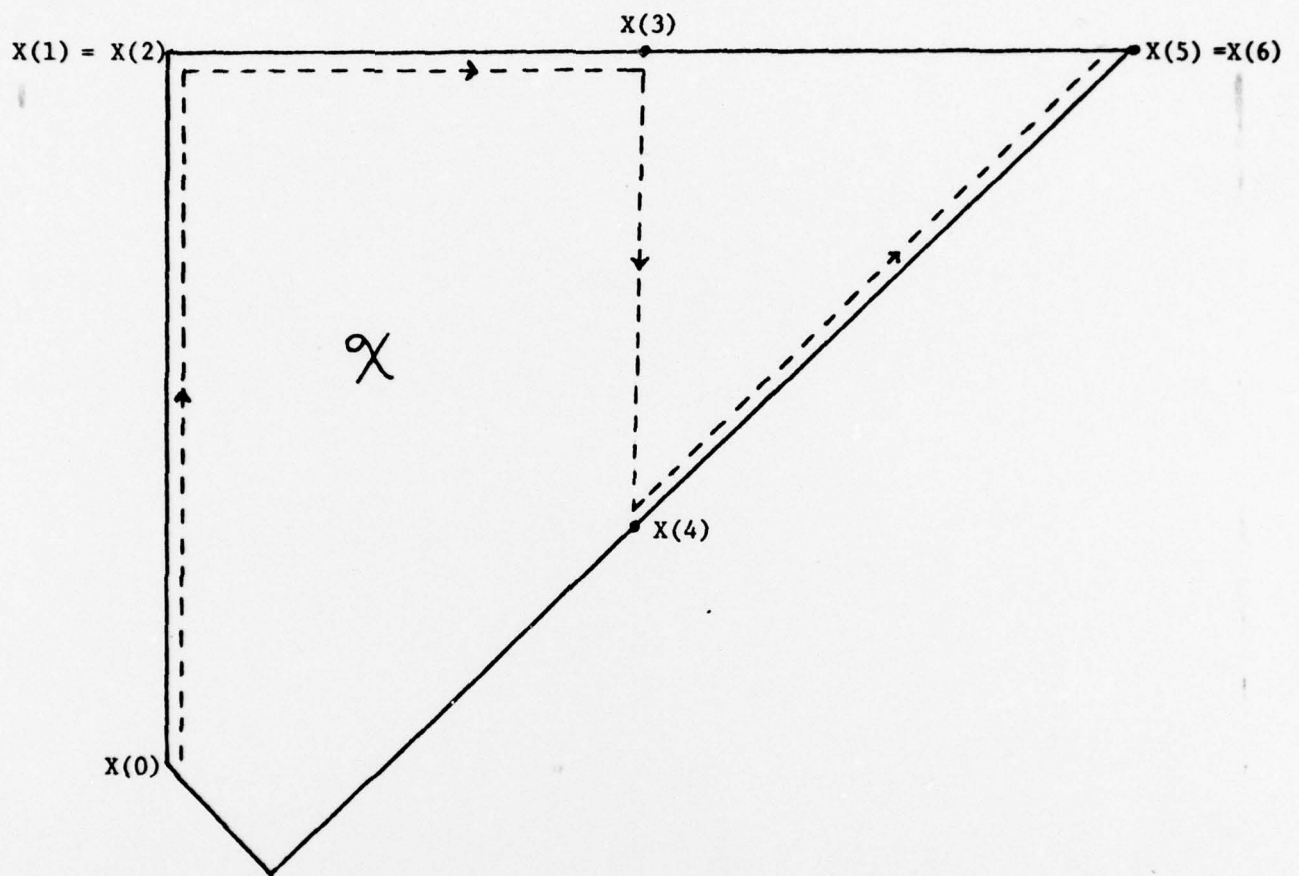| t	X(t)	Θ(t)
0	0, 1	2
1	0, 4	5
2	0, 4	4
3	2, 4	3
4	2, 2	1
5	4, 4	$1\frac{2}{3}$
6	4, 4	0

and X is displayed in Figure 2.

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Figure 2

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After pivoting on positions (2, 3) and (3, 2) of the matrix of (4) and rearranging the variables according to Lemma 2, we get the system (2̃).

6. Appendix

This section demonstrates that the involvement of a degeneracy discussion in Section 2 was necessary in order to cover the possibility that it might contain some zeros.

Consider the application of Lemke's algorithm to the augmented linear complementary problem.

$$Bz + q + p\theta = w$$

$$z \geq 0 \quad w \geq 0 \quad \theta \geq 0 \quad z \cdot w = 0$$

In order to initiate the algorithm one needs (p, q) lexico nonnegative row by row. However, if (p, q) is merely lexico nonnegative and not lexico positive, then Lemke's algorithm may terminate on a ray that is identical to the primary ray after the perturbation is dropped. To illustrate this point, consider the data

$$B = \begin{pmatrix} 0 & 0 \\ -1 & -1 \end{pmatrix} \quad q = \begin{pmatrix} -1 \\ 0 \end{pmatrix} \quad p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

and perturb q to $q_\epsilon = (-1 + \epsilon, \epsilon^2)$.

Lemke's algorithm begins with the solution (w, z, θ) = $(0, \epsilon^2, 0, 0, 1 - \epsilon)$, iterates through the solutions $(0, 0, \epsilon^2, 0, 1 - \epsilon)$ and $(0, 0, 0, \epsilon^2, 1 - \epsilon)$, and terminates with the ray of solutions $(0, 0, 0, \epsilon^2, 1 - \epsilon) + t(1, 0, 0, 0, 1)$ with $t \geq 0$. Upon dropping the perturbation we see that the secondary ray of the perturbed problem becomes the primary ray of the original problem.

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Given a nonempty set X in R^n of form $\{x \geq 0 : Ax \leq a\}$ with an affine function $\beta(x) = Cx + c$ from X to R^n we consider (existence and) computation, in a finite number of steps, of a stationary point. A point x^* in X is defined to be a stationary point of (X, β) if $x \cdot \beta(x^*) \geq x^* \cdot \beta(x^*)$ for all x in X. The existence and computation of stationary points is, in particular, central to the solution of certain quadratic programs, matrix games, and economic equilibrium problems.